27-12-2011 303

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Faculty of Engineering

Electrical and Electronics Engineering Department

EE 303 Numerical Techniques and Programming

Midterm I, November 2nd, 2009

a) Answer all questions to the best of your knowledge.
b) Show all steps and carry all calculations up to 4 digits unless otherwise mentioned.
c) No question will be answered during the exam.
d) Time allowed: *2 hours*

(a) Find Taylor series expansion. for the function $\ln(x)$
(b) Derive a recursive formula for the series in part(a)
(c) What is the relative error of 5th order Taylor series expansion of $f(1.1)$ centered around x=1.0? (true value=0.09531)
(d) Write a c program for computing the sum of the series. Your program should continue computing until the term value drops down to 10^{-9}
(e) **Define the following:** ill-condition matrix, Singular matrix, partial and full pivoting, Matrix augmentation.

(10 Marks)

Q2- Solve the system

$$\begin{aligned} 2.51\,x + 1.48\,y + 4.53\,z &= 0.05 \\ 1.48\,x + 0.93\,y - 1.30\,z &= 1.03 \\ 2.68\,x + 3.04\,y - 1.48\,z &= -0.53 \end{aligned}$$

(a) Using Gaussian elimination with out partial pivoting and rounding to 4 decimal places.
(b) Repeat part (a) with partial pivoting and digits chopping after 3 significant figures.
(c) What is the *residual* of the answers obtained in parts (a) and (b) of this question?

(10 Marks)

Q3- For the following system

$$\begin{bmatrix} 0.5x & 1.5 \\ 0 & 2.1 \end{bmatrix}$$

(a) Find the value of x that will make the condition number of the system $\simeq$100 Your approximation should have error no greater than 0.001 .

(10 Marks)

Good luck to all of you

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2)

a) $f(x) = \ln(x)$

$f'(x) = \frac{1}{x}$, $f''(x) = -\frac{1}{x^2}$, $f^{(3)}(x) = \frac{2}{x^3}$, $f^{(4)}(x) = -\frac{6}{x^4}$

odd derivative positive
even " -ve $\Rightarrow (-1)^{n+1}$

Numerator
$n=1 \rightarrow 1$
$n=2 \rightarrow 1$
$n=3 \rightarrow 2 = (3-1)!$
$n=4 \rightarrow 6 = (4-1)!$
$= (n-1)!$

$$f^{(n)}(x) = \frac{(-1)^{n+1} \cdot (n-1)!}{x^n}$$

$$\therefore f(x_{i+1}) = f(x_i) + \sum_{n=1}^{\infty} \frac{f^{(n)}(x) \cdot h^n}{n!}$$

$$= f(x_i) + \sum_{n=1}^{\infty} \frac{(-1)^{n+1} \cdot (n-1)!}{x^n} \cdot \frac{h^n}{n!}$$

$\because \frac{(n-1)!}{n!} = \frac{1}{n}$

$$\boxed{\therefore f(x_{i+1}) = \ln(x_i) + \sum_{n=1}^{\infty} \frac{(-1)^{n+1} \cdot h^n}{n \cdot x^n}}$$

$T_0 = \ln(x_i)$

$T_1 = T_0 + \frac{h}{x}$

$T_2 = T_0 + T_1 + (-1)^{n+1} \cdot \frac{h^2}{2x^2}$

$T_n \neq (\sim) T_{n-1}$

because $\underbrace{T_0}_{\ln}$ differ from $\underbrace{T_{n-1}}_{h^2}$

c) at $n=5$, $x_i = 1$, $x_{i+1} = 1.1 \Rightarrow h = 0.1$

$$f(1.1) \cong \ln(1) + \sum_{n=1}^{\infty} \frac{(0.1)^n}{n}$$

$$\cong 0 + 0.1 - \frac{(0.1)^2}{2} + \frac{(0.1)^3}{3} - \frac{(0.1)^4}{4} + \frac{(0.1)^5}{5}$$

Round off at 7 digits

$f(1.1) \cong 0.0953103$

$$\%\varepsilon_t = \left| \frac{0.0953102 - 0.0953103}{0.09531} \right| \times 100 = 3.15 \times 10^{-4}$$

d) The condition of control of loop; Term value drops down to 10^{-9}

It doesn't mean that $f(1.1) < 10^{-9}$

It mean that either $(T_v - \text{Estimated}) < 10^{-9}$

or $\left(\frac{T_v - \text{Estimated}}{T_v} \right) < 10^{-9}$ $< 10^{-7}\,\%$

or $\left(\text{Current Estimated} - \text{Perevious Est} \right) < 10^{-9}$

or $\left(\frac{\text{〃} \quad \text{〃} - \text{〃}}{\text{Current Estimated}} \right) < 10^{-9}$ or $< 10^{-7}\,\%$

Rounding off for 10 digites at least

~~to~~ Taking $E_r = |T_v - \text{Estimated}|$

```
#include<stdio.h>
#  "  <math.h>
#  "  <conio.h>
main( ){
int i=0; float xi, x, fofx, Tv, Er, Error;
Printf(" \n  xi  = "); scanf(" %.11f", &xi);
  "  (" \n  x = "); scanf(" %.11f", & x);
rintf(" \n Absolute Error = "); scanf(" %.11f", Error);
Tv = log(x);      // ln(x) in C or MATLAB written log(x)
~~= fabs(x-xi);~~
 = fabs(x-xi);
~~of(x)~~
-ofx = ln(xi)
Er = fabs(Tv- fofx);    f(x)
Printf(" \n  Iteration     f(x)    Absolute Error ");
Printf(" \n    %d      %.11f      %.11f", i, fofx, Er);
while( Er >= Error)

=i+1
fx = fofx+(pow(-1,i+1)* pow(h,i)/(i * pow(xi, i)));
r = fabs(Tv- fofx);

tchc();
```

hat detrminatt equal to zero

e) ill condition matrix ≙ It is determinant near singular

Singular Matrix ≙ It is a matrix has a determinant equal to zero

Partial pivoting ≙ Exchang rows or columns to avoid division by zero pivoting element or to avoid " " very small pivoting element

Full pivoting ≙ Exchang both rows & columns only

Matrix Augmentation ≙ putting the system cofficients and the vector column of the R.H.S of a linear system of eq in one Matrix
To reduce Round-off errors

$$a_{11} x_1 + a_{12} x_2 + a_{13} x_3 = b_1$$
$$a_{21} x_1 + a_{22} x_2 + a_{32} x_3 = b_2$$
$$a_{31} x_1 + a_{32} x_2 + a_{33} x_3 = b_3$$

rows

Calumns only

$$\begin{bmatrix} a_{11} & a_{12} & a_{13} & b_1 \\ a_{21} & a_{22} & a_{23} & b_2 \\ a_{31} & a_{32} & a_{33} & b_3 \end{bmatrix} \rightarrow \text{Augmented Matrix}$$

both rows an & columns

Q2

(a) Without pivoting

$$2.51x + 1.48y + 4.53z = 0.05$$
$$1.48x + 0.93y - 1.3z = 1.03$$
$$2.68x + 3.04y - 1.48z = -0.53$$

$$\begin{bmatrix} 2.51 & 1.48 & 4.53 & 0.05 \\ 1.48 & 0.93 & -1.3 & 1.03 \\ 2.68 & 3.04 & -1.48 & -0.53 \end{bmatrix}$$

Ex

has a determinal equel to zero
near singular

limination of x from R_2 & R_3

$2 \leftarrow R_2 - f_{12} \cdot R_1$

$3 \leftarrow R_3 - f_{13} R_1$

$_2 = \frac{a_{21}}{a_{11}} = \frac{1.48}{2.51} \Rightarrow f_{12} = 0.5896$

1.48

$$\begin{bmatrix} 2.51 & 1.48 & 4.53 & 0.05 \\ 0 & 0.0573 & -3.971 & 1.001 \\ 0 & 1.46 & -6.[illegible]17 & -0.0004 \end{bmatrix}$$

Eliminating y *from* R_3

$$f_{23} = \frac{a'_{32}}{a_{22}} = \frac{1.46}{0.0573} = 25.48$$

$$R''_3 = R'_3 - f_{23} \cdot R'_2$$

$$\begin{bmatrix} 2.51 & 1.48 & 4.53 & 0.05 \\ 0 & 0.0573 & -3.971 & 1.001 \\ 0 & 0 & 94.86 & -25.51 \end{bmatrix}$$

$$94.86z = -25.51 \Rightarrow \boxed{z = -0.2689}$$

$$0.0573y - 3.971z = 1.001$$

$$\therefore y = \frac{1.001 + 3.971 \times (-0.2689)}{0.0573}$$

$$\boxed{\therefore y = -1.166}$$

$$2.51x + 1.48y + 4.53z = 0.05 \Rightarrow x = \frac{0.05 - 1.48y - 4.53z}{2.51}$$

$$\boxed{x = 1.193}$$

Substitute x, y, z in the original eq$_s$

$0.0504 \cong 0.05$

$1.031 \cong 1.03$

$\underbrace{0.0504}_{\text{too far}} \ncong -0.53$???

b) With paritial pivoting & chopping after 3 digits

$R_3 \Longleftrightarrow R_1$

$$\begin{bmatrix} 2.68 & 3.04 & -1.48 & -0.53 \\ 1.48 & 0.93 & -1.3 & 1.03 \\ 2.51 & 1.48 & 4.53 & 0.05 \end{bmatrix}$$

Eliminating x from R_2 & R_3

$f_{12} = \frac{1.48}{2.68} = 0.552$, $f_{13} = \frac{2.51}{2.68} = 0.936$

$$\begin{bmatrix} 2.68 & 3.04 & -1.48 & -0.53 \\ 0 & -0.748 & -0.483 & 1.323 \\ 0 & -1.365 & 5.915 & 0.546 \end{bmatrix}$$

Eliminating y from R_3

$\cancel{f_{23} = 1.365}$ $\rightarrow$ Another pivoting $R_2' \Longleftrightarrow R_3'$

$$\begin{bmatrix} 2.68 & 3.04 & -1.48 & -0.53 \\ 0 & -1.365 & 5.915 & 0.546 \\ 0 & -0.748 & -0.483 & 1.323 \end{bmatrix}$$

$f_{23} = \frac{0.748}{1.365} = 0.548 \Rightarrow \begin{bmatrix} 2.68 & 3.04 & -1.48 & -0.53 \\ 0 & -1.365 & 5.915 & 0.546 \\ 0 & 0 & -3.724 & 1.024 \end{bmatrix}$

$\therefore -3.724z = 1.024$

$\therefore z = -0.274$

$-1.365y + 5.915(-0.274) = 0.546 \Rightarrow y = \frac{0.546 + 5.915 \times 0.274}{-1.365}$

$y = -1.597$

$2.68x + 3.04y - 1.48z = -0.53 \Rightarrow x = \frac{-0.53 + 1.48z - 3.04y}{2.68}$

Substituting in the original eq.

$51(1.\cancel{5}451) + 1.48(-1.587) + 4.53(-0.274) = 0.051 \cong 0.05$

$.48(1.451) + 0.93(-1.587) - 1.3(-0.274) = 1.028 \doteq 1.03$

$2.68(1.451) + 3.04(-1.587) - 1.48(-0.274) = -0.531 \cong 0.53$

So Eventhough we chopped after 3 digits, the values of x, y, z are very close to the exact values, this is because the use of partial pivoting reduces the round off error results in ~~the~~ sec (b)

) Residual

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} - \begin{bmatrix} \bar{x} \\ \bar{y} \\ \bar{z} \end{bmatrix}$$

$|x - \bar{x}| = \big|1.193 - 1.451\big| = 0.258$

$|y - \bar{y}| = |-1.166 - (-1.587)| = 0.421$

$|z - \bar{z}| = |-0.2689 - (-0.274))| = 0.0051$